Quantum Harmonic Oscillator

Hamiltonian for harmonic oscillator:

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}kx^2 ,$$

Where k – force constant

Then Schrödinger equation $\hat{H} = E\psi$ for harmonic oscillator can be written as:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \frac{1}{2}kx^2 = E\psi$$

Energy levels are equidistant:

$$E_n = (n + \frac{1}{2})\hbar\omega ,$$

or

$$E_n = (n + \frac{1}{2})h\nu$$

Where *n=0, 1, 2,…,* $\omega = \sqrt{\frac{k}{m}}$ – angular frequency in (rad/sec) or $\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, k – force constant, m – mass of the oscillator.

Angular frequency (ω) in (rad/sec) can be converted into conventional frequency (ν) measured in (sec^{-1}):

$$\nu\ (\text{sec}^{-1}) = \frac{1}{2\pi}\omega\ (\text{rad/sec})$$

Wavefunctions of the harmonic oscillator can be expressed using Hermite polynomials $H_n(y)$:

$$\psi_n(y) = \left(\sqrt{\frac{\beta}{\pi}}\frac{1}{2^n n!}\right)^{1/2} H_n(y)\exp\left(-y^2/2\right),$$

Where $\beta^2 = \frac{mk}{\hbar^2}, y = \sqrt{\beta}x, H_n(y)$ - Hermite polynomials.

Hermite polynomials:

$$H_n(y) = (-1)^n \exp(y^2)\frac{d^n \exp(-y^2)}{dy^n}$$