# Integration of the Elementary Functions, Antiderivative of a Function

Integration is an operation inverse to differentiation:

$$f(x) \xrightarrow{\text{differentiation}} f'(x)$$
$$f'(x) \xrightarrow{\text{integration}} f(x)$$

Indefinite integral of a function $f(x)$ denoted as: $\int f(x)dx$
Indefinite integrals of some elementary functions are listed in Table 2.

**Table 2. Indefinite integrals of the elementary functions** $f(x)$, $c = const$, $\alpha = const$

| $f(x)$ | $\int f(x)dx$ |
|---|---|
| $\int dx$ | $x + c$ |
| $\int x^{\alpha} dx, \alpha \neq -1$ | $\frac{x^{\alpha+1}}{\alpha+1} + c$ |
| $\int \frac{dx}{x}$ | $\ln\lvert x\rvert + c$ |
| $\int \sin(x)dx$ | $-\cos(x) + c$ |
| $\int \cos(x)dx$ | $\sin(x) + c$ |
| $\int e^{x} dx$ | $e^{x} + c$ |

# Definite integral

Definite integral of a function $f(x)$ denoted as: $\int_a^b f(x)dx$. Value of the definite integral $\int_a^b f(x)dx$ is a number $S$ which can be calculated using fundamental theorem of calculus Newton-Leibnitz equation:

$$S = \int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a),$$

where $F(x)$ is the indefinite integral (antiderivative) of the function $f(x)$

# Rules for Definite Integrals

$$\int_a^b (f(x)+g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$$

$$\int_a^b (f(x)-g(x))dx = \int_a^b f(x)dx - \int_a^b g(x)dx$$

$$\int_a^b cf(x)dx = c\int_a^b f(x)dx,\ c = const$$

$$\int_a^b f(cx)dx = \frac{1}{c}\int_{ac}^{bc} f(x)dx,\ c = const$$

Substitution (Composition Rule or Chain Rule)

$$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(x)dx$$

Product Rule (Integration by Parts)

$$\int_a^b f(x)g'(x)dx = f(x)g(x)\Big|_a^b - \int_a^b g(x)f'(x)dx$$

# Appendix

**Table 3. Values of sin and cos functions at various angles**

| θ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Deg. | 0° | 30° | 45° | 60° | 90° | 180° | 270° |
| | Rad. | 0 | $\pi/6$ | $\pi/4$ | $\pi/3$ | $\pi/2$ | $\pi$ | $3\pi/2$ |
| sin(θ) | | 0 | $1/2$ | $1/\sqrt{2}$ | $\sqrt{3}/2$ | 1 | 0 | -1 |
| cos(θ) | | 1 | $\sqrt{3}/2$ | $1/\sqrt{2}$ | $1/2$ | 0 | -1 | 0 |